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EXPLICIT VERSUS IMPLICIT NUMERICAL IMPLEMENTATIONS OF THE WALL SLIP BOUNDARY CONDITION

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Contraction flow, wall slip, vortex dimension, profile extrusion

Abstract

A new approach for the implementation of linear Navier slip boundary conditions into a finite volume method is presented. The details of this implementation are given for a simple geometry and using Cartesian coordinates. A comparison is made between this new method (implicit approach) and the usual iterative process (explicit approach). It could be found that for this new implementation the convergence issues during the iterative procedure are solved and for some specific geometry the greater the slip velocity better is convergence of the process. With this new arrangement the boundary conditions are implicit in the system of equations and there is no need for any relaxation factor in the slip velocity when using high slip (friction) coefficients. The robustness of the code was also tested in a 4:1 contraction, allowing the study of the vortex dimension with slip velocity, using a Newtonian and PPT constitutive equations.

Introduction

The motion of incompressible fluids is governed by the the Navier Stokes Equations (Batchelor 1967), which express the conservation of mass and momentum. In 2D this equations are the following:

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \quad \textbf{continuity} \qquad (1.1)$$

$$\underbrace{\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)}_{x-\textbf{momentum}} \qquad (1.2)$$

$$\underbrace{\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(\rho vv)}{\partial y}=-\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)}_{y-\textbf{momentum}} \qquad (1.3)$$

Assuming that the fluid slips at the wall, and the slip velocity is given by the *Navier Slip* Law (Navier 1827) the following relationship, between slip velocity (u_s) and the wall shear stress (τ_{xy}), applies:

$$u_s=\left[-\textbf{sign}\left(\tau_{xy}\right)\right]b.\tau_{xy}$$
$$\Leftrightarrow \underbrace{u_s\left[-\textbf{sign}\left(\partial u/\partial y\right)\right]k\left(\partial u/\partial y\right)}_{Couette\ \ flow} \qquad (1.4).$$

Numerical implementation of wall slip

Explicit approach

Using the usual explicit approach for the implementation of the wall slip boundary condition the calculation proceeds as illustrated in the flow chart of Figure 1, until convergence is achieved.

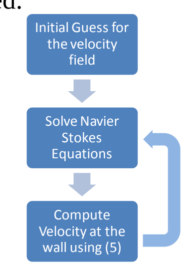

Figure 1. Explicit implementation of the wall slip.

However, the explicit approach requires the employment of high levels of relaxation (Sunarso and Yamamoto 2007) increasing with the intensity of slip imposed, fact that delays the calculation, and may limit the maximum level of slip imposed.

Implicit approach

Since most of the terms in the conservation of momentum equations are not influenced by the existence of slip at the wall, an alternative approach for the implementation of this boundary condition can be proposed. For the sake of simplicity one can consider a flow between parallel plates as illustrated in Figure 2.

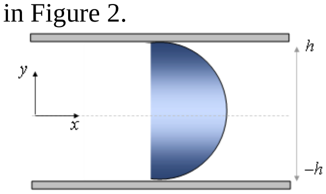

Figure 2. Flow and geometry.

For this geometry and analyzing in detail a computational cell close to the upper wall, only the diffusive term in the x-momentum equation is affected by the existence of a slip velocity. In this cell, the variation of u along the y direction must be calculated. The discretization of this term in the computational cell shown in Figure 3, gives:

$$\left(\partial u / \partial y\right)_{wall} \simeq (u_n - u_p)/\Delta y_f \qquad (1.5)$$

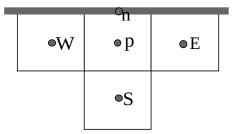

Figure 3. Computational cell near the wall.

Using the Navier Slip Law (1.4), an explicit relation between u_n and u_P can be obtained:

$$u_n = -k\frac{u_n - u_p}{\Delta y_f} \Leftrightarrow u_n = \left(\frac{k/\Delta y_f}{k/\Delta y_f + 1}\right).u_p \qquad (1.6)$$

Thus the derivative of the velocity at the wall, Eq. (1.5), can be approximated by:

$$\left(\frac{\partial u}{\partial y}\right)_{wall} \simeq \frac{\left((k/\Delta y_f)/(k/\Delta y_f + 1)\right).u_p - u_p}{\Delta y_f} \qquad (1.7)$$

Consequently, there is no need to obtain the value of u_n to solve the system of equations, since the required term is only function of u_P.

Using this formulation, during the iterative calculation process, the last step of the flow chart shown in Figure 1 does not have to be performed. The slip velocity can be obtained, at the end of the calculation procedure, using Eq. (1.7).

Results and Discussion

The two alternative approaches described for the implementations of the wall slip boundary condition, were tested in a simple problem of the flow of a Newtonian Fluid between parallel plates, as shown in Figure 2. The results obtained, in terms of iterations required to obtain convergence are illustrated in Figure 4.

The resolution with the explicit approach requires, in general, more iterations than the implicit approach, since the relaxation factor, needed to obtain convergence, has to be increased with the level of slip. Moreover, for the largest levels of slip, S1 and S0, it was not possible to reach a converged solution with the explicit formulation.

To test the robustness of the implicit procedure, a 4:1 contraction was also used (see Fig. 5). It was possible to obtain convergence both with Newtonian and a PTT fluid for high slip velocities. This allowed studying the evolution of the vortex geometry with the increasing if the slip velocity. For Newtonian fluids the vortex dimension is decreasing with an increase in slip (see Fig. 6-top). For non-Newtonian fluids the vortex dimension changes little (see Fig. 6-bottom).

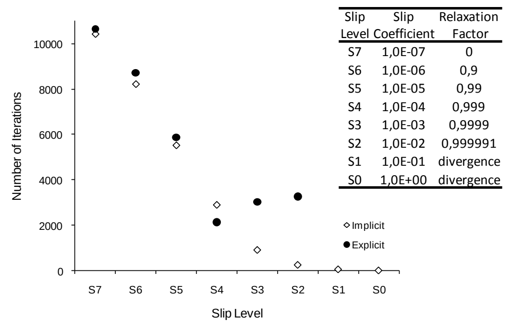

Figure 4. graph representing a comparison for the number of iterations needed to get convergence using the two methods.

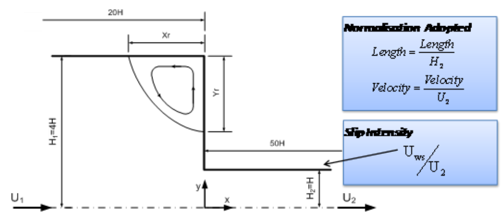

Figure 5. Variables rescaling in the 4:1 contraction.

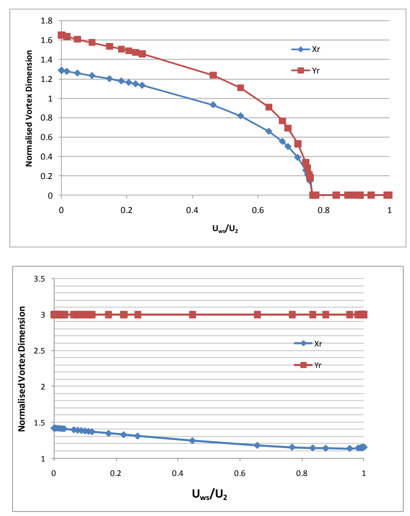

Figure 6. Evolution of vortex dimension in Newtonian fluid (top) PTT fluid (bottom).

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References

Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge University Press, Cambridge, UK, 1967.

Navier C. L. M. H., Mem. Acad. R. Sci. Inst. Fr, 6 389-440, 1827.

Sunarso, A., Yamamoto, T., Mori, N., Numerical Analysis of Wall Slip Effects on Flow of Newtonian and Non-Newtonian Fluids in Macro and Micro Contraction Channels, J. Fluids Eng. 129, 23 (2007).