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A BIOMECHANICAL MULTIBODY FOOT MODEL FOR FORWARD DYNAMIC ANALYSIS FOR ORTHOSIS DESIGN

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ABSTRACT

The main purpose of this work is to present a three-dimensional biomechanical multibody foot model suitable to perform forward dynamic analysis for orthosis design. The proposed approach takes into accounts the different contact phenomena that develop between the foot and ground, namely the geometric, kinetic, dynamic and material properties of the foot-ground interface during the stance phase of the human gait. The interaction between the foot and ground bodies is provided by the introduction of a set of spheres under the plantar surface of the foot. In the sequel of this process, a general mathematical methodology for contact detection between the foot and ground surfaces is presented. Then, in a simple way, when the foot-ground contact occurs, appropriate constitutive laws for contact phenomena are applied. These laws take into account the vertical ground reaction force as well as the friction phenomena, namely the Coulomb and viscoelastic friction effects. Finally, the results obtained from computational and experimental analysis are used to discuss the main assumptions and procedures adopted through this work.

INTRODUCTION

Over the last few years, a good number of researchers have studied biomechanical models, based on multibody dynamics, in a wide range of different applications where the human motion is characterized by gross motions (Silva et al. 1997, Tagawa et al. 2000, Nazer et al. 2008). However, most of them do not consider the foot-ground interaction, because they are developed within the frame work of inverse dynamic analysis formulation (Kraus et al. 2005, Wit and Czaplicki 2008, Tlalolini et al. 2010). This paper is concerned with the development of a three-dimensional foot model that can be used to perform forward dynamic analysis associated with the human motion. The model is developed under the framework of multibody systems methodologies and natural coordinates are used to describe the system components and the kinematic joints. The foot and ground bodies are modeled as contacting elements in which their characteristics are function of the geometric and material properties of the surfaces. The equations governing the dynamical behavior of the general system incorporate the normal and tangential forces due to the contact of the plantar surface and ground. A continuous contact model, in which the local deformation and contact forces are treated as continuous, provides the normal forces. For this model it is assumed that material compliance and damping parameters are available. The dry and viscoelastic friction effects are also considered. The model proposed here is straightforward and computationally efficient.
It is known that the human foot is a complex multi-articular mechanical structure composed by bones, articulations and soft tissues, which are controlled by both intrinsic and extrinsic muscles (Abboud 2002). This idea is corroborated by Sereig and Arvikar (1989) who defined the foot as a multi-segmented and highly ligamentous structure articulated at numerous joints and assisted by a meticulous arrangement of extrinsic and intrinsic muscles that provide a diverse range of motions and functions. The foot is a vital part of the human locomotion apparatus, not only because it supports the weight of the complete human body, but also because during human gait the body pivots about it. The foot also forms an important kinetic and kinematic boundary condition between the model and the ground. In a broad sense, most of the existing biomechanical foot models consider the foot as single rigid body, neglecting the multi-segmental and deformable structures that it comprises. In general, these models are used to represent the foot-ground interaction that occurs

during the stance phase of human gait. There are other simple approaches that treat the ankle joints directly connected to the ground, and, consequently, disregarding the existence of the foot (Siegler et al. 1982). Other modeling foot approach is to consider the foot kinematically fastened to the ground during the stance phase of gait in order to limit and control the motion (Pandy and Berme 1989, Yang et al. 1990, Chou et al. 1995).

One of the first biomechanical foot models that does not include any kinematic conditions to the movement of the feet was developed by Meglan (1992). In this model, the interaction of the plantar surface with the ground is described by viscoelastic elements. The mechanical properties of the viscoelastic elements were identified using experimental data on heel pads (Valiant 1984). The shear forces were calculated using a modified Coulomb friction model. However, besides the existence of some experimental measurement errors, this model leads to some numerical difficulties, namely in what concerns the solution of the equations of motion. Hence, the computational simulations failed due to stiffness problems in the numerical solution of the equations of motion.

Another foot model that describes the foot-ground contact interaction during the stance phase of gait using viscoelastic elements was introduced by Gilchrist and Winter (1996). In this three-dimensional model, the foot was described as a two segment model in which the metatarsal-phalangeal articulation was represented by one revolute joint. This foot model was developed and used to simulate the foot global motion, from heel contact to toe-off. The viscoelastic contact properties were described by a collection of 9 vertically oriented spring-damper elements, located along the midline of the foot. Associated with each vertical spring-damper element, two orthogonally horizontal dampers were considered with the purpose to account for the friction components of the ground reaction force. In addition, torque acting on the metatarsal-phalangeal articulation was included as a torsional spring-damper element. In this work, the stiffness and damping characteristics were obtained by using trial and error approach. Although the results of the simulation with this foot model reproduced well the kinematics and kinetics, it was shown that the model is quite sensitive to the stiffness and damping parameters.

Güller et al. (1998) used a sphere model to represent the plantar surface of the foot during locomotion simulations. The mechanical properties of the sphere were those published in reference (Valiant 1984). The heel pad is modeled as viscoelastic spherical body compressed against a rigid plane. The stiffness and damping properties of the heel pad tissue were represented by Kelvin-Voigt elements, in which the spring and damper are in parallel. It should be highlighted that this model is found to be insensitive to variations in stiffness and damping parameters.

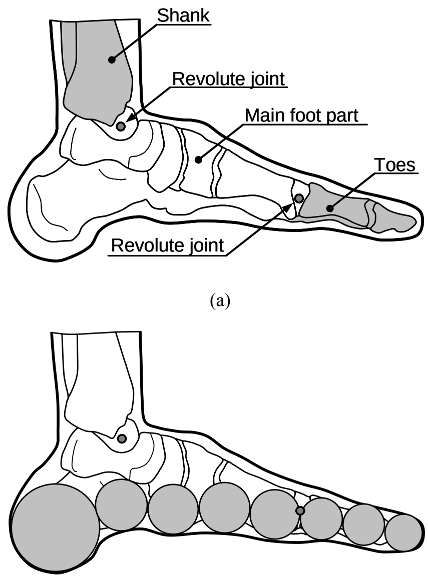
Neptune et al. (2000) developed a three-dimensional musculoskeletal foot model combining the equations of motion for the musculoskeletal system and model for the muscle force generation and ground contact elements. This musculoskeletal model consisted of rigid segments representing the rear-foot, mid-foot, toes, talus, shank, patella, thigh of the supported leg, pelvis and the rest-of-body segment. The contact between the foot and the ground was modeled by 66 discrete independent viscoelastic elements, each attached to one of the three foot segments in locations that describe the three dimensional exterior surface of a shoe when the foot joints are in a neutral position. The obtained results showed that the ground reaction force was insensitive to the variations of the shoe stiffness.

A general biomechanical model, that also includes the foot-ground interaction, was presented by Anderson and Pandy (2001). The skeleton was represented by a 10 anatomical segments that results in 23 degree-of freedom. The pelvis was modeled as a single rigid body and the remaining segments branched in an open chain from the pelvis. The head, arms and torso (HAT) were lumped into a single rigid body, being this segment articulated with the pelvis. Two elements were used to model each foot, that is, the hindfoot and the toes anatomical segments. The interaction of the feet with the ground was simulated using a series of spring-damper elements distributed under the plantar surface of each foot. Some differences between numerical and experimental results were observed. Spikes in the vertical ground reaction force just prior to opposite toe-off are caused by very small-amplitude oscillations in the foot springs. The vertical force does not fall to zero at the end of stance phase, which results from a delay in weight transfer onto the contralateral leg at the beginning of double support. A computational simulation of human walking was also described by Wojtyra (2003). The biped model consists of 8 rigid bodies. The trunk was modeled as two bodies connected by a revolute joint. Compliance laws were used to model the contact and friction forces developed in the foot-ground interaction. Then, a set of 5 force vectors

acting on each foot was used to model ground reaction forces. In this model, the number of forces and points of their applications were chosen to obtain an adequate approximation of reality. This model was successfully validated, being the results of simulation in reasonably good agreement with experimental measurements.
More recently, Peasgood et al. (2007) presented a general two-dimensional multibody model for dynamic walking gait analysis. In this work, a foot contact model was created using a point contact force at the heel and ball of each foot. When a contact condition is detected, the vertical contact force is computed on the foot using a nonlinear spring-damper element. The horizontal force at the contact point was modeled using the Coulomb friction force model. This foot model was used to support the development of a feedback control system that stabilizes the torso orientation during a human walking gait dynamic simulation, enabling arbitrarily long simulations. Millard and co-authors (2007) used the multibody mechanical model developed by Peasgood et al. (2007) to introduce a new foot model. A predictive forward dynamic simulation of human gait was performed for a multi-step analysis. Foot contact forces were calculated using a 2-point foot contact model, with a contact located at the heel and metatarsal. This foot contact model produced ground reaction forces that substantially differ from those observed during the normal gait (Valiant 1984). The authors showed that the poor performance of the foot contact model was partly responsible for the joint torque differences between healthy human gait and the simulated results.

MATHEMATICAL FOOT MODEL

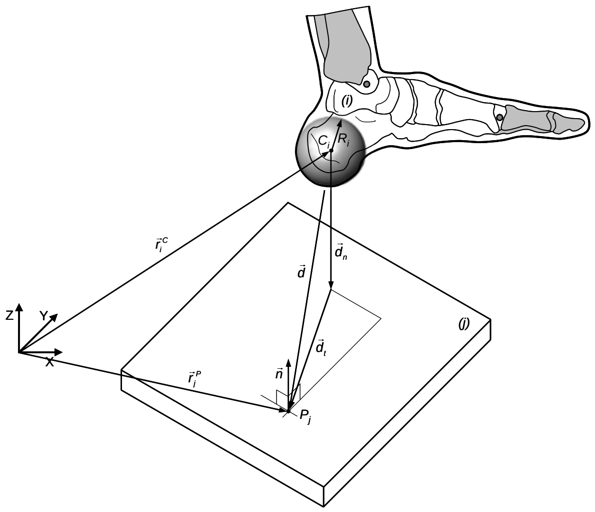
In order for the foot contact model to be used, it is first necessary to develop a mathematical model for foot and ground surfaces in the biomechanical multibody system. The biomechanical multibody foot model proposed in this study is composed by 3 rigid bodies that represent the shank, the main foot part and the toes, as it is illustrated in Fig. 1a. The ground is the fourth body, which is rigid and flat. The foot parts are constrained by 2 revolute joints. In the present work, the main foot part and the toes are represented by a set of spheres, as depicted in Fig. 1b. The number of spheres, their radius and their locations can vary and be adjusted with the intent to obtain a better representation of the plantar surface. Furthermore, these characteristics must also taken into account the anatomical and biomechanics of the actual foot, that is, the spheres must be located in the areas of the foot that are most relevant in human gait (Moreira et al. 2009).
The interaction between the foot and the ground is performed by evaluating, at each time step, the potential contacts between the spheres and the rigid flat surface, which depends on the global bodies' position during the simulation. The occurrence of any penetration is used as the basis to develop the procedure to evaluate the local deformation of the bodies in contact. The normal and tangential contact forces are function of the material properties of the contact bodies, penetration and relative contact velocities in both normal and tangential directions.



(b)

Figures 1: (a) Foot parts; (b) Foot plantar surface defined as a set of spheres

In the present work the interaction between the foot and the ground is provided by the introduction of 9 spheres (6 under the plantar surface and 3 under the toes), located along the midline of the foot, as it is schematically shown in Fig. 1b. A general procedure based on the kinematic configuration of the multibody system's bodies is developed with the purpose to detect what are the contacting elements at each instant of the analysis. Figure 2 shows the ground body, denoted by symbol j, which represents a flat rigid body defined by a point P_j and a vector **n** normal to the plane. In order to keep the analysis simple, only one sphere is considered to represent part of the foot denoted as body *i*. The radius of the spheres is R_i, while its center is represented by point C_i. The global coordinate system is indicated by *XYZ*.



Figures 2: Localization vectors of the main geometric elements necessary for contact detection

From Fig. 2 the distance vector **d** between points C_i and P_j can be expressed by

$$\mathbf{d} = \mathbf{r}_j^P - \mathbf{r}_i^C \tag{1}$$

where both $\mathbf{r}_j^P$ and $\mathbf{r}_i^C$ are described in global coordinates with respect to the inertial reference frame (Nikravesh 1988). Taking into account that the normal of the ground plane must be expressed in terms of global coordinates, the components of vector **d** in the normal and tangential to the ground can be evaluated as

$$\mathbf{d}_n = (\mathbf{d}^T\mathbf{n})\mathbf{n} \tag{2}$$

$$\mathbf{d}_t = \mathbf{d} - \mathbf{d}_n \tag{3}$$

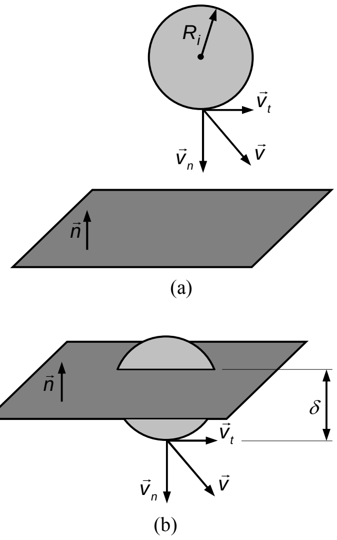
These two vectors are represented in Fig. 2. Thus, the geometric condition used to check if the sphere and plane are in contact is given by

$$\delta = R_i - d_n \tag{4}$$

in which R_i is the sphere radius and d_n denotes the magnitude of the vector $\mathbf{d}_n$, that represents the projection of vector **d** onto the normal direction **n**

$$d_n = \mathbf{d}_n^T\mathbf{d}_n = \mathbf{d}^T\mathbf{n} \tag{5}$$

By analyzing Eq. (4) it can be observed that relative penetration between ground and sphere occurs when δ is greater than zero, otherwise the bodies are separated from each other. These two scenarios are represented in Fig. 3, where the relative normal and tangential velocities are also illustrated.



Figures 3: (a) Approaching phase between a sphere and a plane; (b) Contact between a sphere and a plane, where the pseudo-penetration, δ, is visible

In most of the relevant contact force models, it is of paramount importance to evaluate the dissipative effect that takes place during the contact process. In such models, it is necessary to calculate the relative velocity of the contacting surfaces in both normal and tangential directions. The normal relative velocity determines whether the contact bodies are approaching or separating. Similarly, the tangential relative velocity helps in the determination if the contact bodies are sliding or sticking (Flores et al. 2006). The relative scalar velocities, normal and tangential to the plane of collision, are found by projecting the relative contact velocity onto each one of these directions, yielding

$$\mathbf{v}_n = (\mathbf{v}^T\mathbf{n})\mathbf{n} \tag{6}$$

$$\mathbf{v}_t = \mathbf{v} - \mathbf{v}_n \tag{7}$$

in which the relative velocity between the points C_i and P_j is given by

$$\mathbf{v} = \mathbf{v}_j^P - \mathbf{v}_i^C \tag{8}$$

In short, if the contact between sphere and ground is effective, then the pseudo-penetration is given by Eq. (4), being the pseudo-velocity of penetration is

$$\dot{\delta} = \|\mathbf{v}_n\| \tag{9}$$

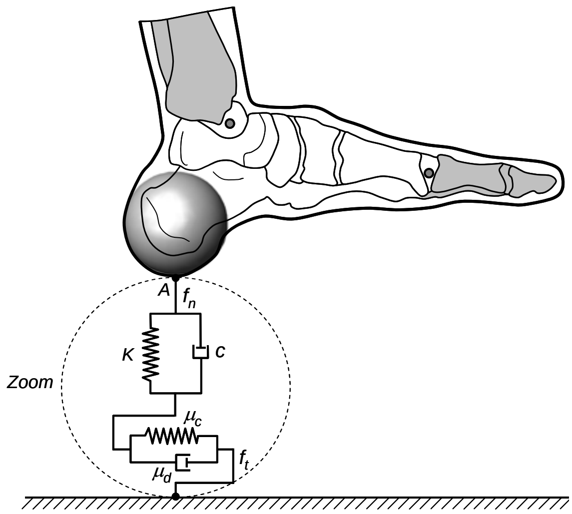
CONSTITUTIVE EQUATIONS FOR CONTACT

With the purpose to evaluate efficiently the contact forces resulting from the foot-ground interaction, special attention must be given to the numerical description of the contact force model. Information on the contact velocity, material properties of the colliding bodies and geometric characterization of the contacting surfaces must be included into the contact force model. These characteristics are observed with a continuous contact force law, in which the deformation and contact force is considered as a continuous function (Lankarani and Nikravesh 1990). Furthermore, it is important that the contact force model can be added to the stable integration of the multibody system's equations of motion. In a broad sense, any contact problem in multibody dynamics can be divided into three main steps, namely (*i*) definition of the geometric properties; (*ii*) development of a methodology for contact detection; (*iii*) application of appropriate constitutive laws for contact forces that develop in the normal and tangential directions (Hippmann 2004). In what follows, these issues are presented and discussed for the case of foot-ground interaction during human gait.

In this study, as it was already described in the previous section, the contact problem between the foot and ground is defined as the contact situation between a set of spheres and a flat surface. The elements in contact are assumed to be rigid, however the contacts are deformable, materialized by an overlap or pseudo-penetration. The problem of contact detection deals with the evaluation of the minimum distance between potential contacting points, at every time step of simulation. This is of paramount importance when the multibody system has multiple contact points, as it is the case of foot-ground interaction. In a simple way, the mathematical expression for that contact involves sphere radius and bodies positions and allows for the calculation of pseudo-penetration of the bodies. The constitutive laws that represent the physical interaction between foot and ground are functions of the relative penetration. The best known force model for the contact between two spheres of isotropic materials was developed by Hertz, which is based on the theory of elasticity, (Hertz 1882)

$$\mathbf{f}_n = K\delta^m \mathbf{n} \tag{10}$$

where the parameter K represents the relative contact stiffness that depends on the geometric and material properties of the contacting bodies, δ the relative penetration given by Eq. (4), m a non-linear coefficient and $\mathbf{n}$ is the normal unit vector of the contact plane. This is a purely elastic model, that is, it does not account for the energy dissipation during the contact process. In fact, one of the most complex tasks when modeling any contact deals with the issue associated with the process of energy dissipation. Therefore, in the present work, the normal contact force between the foot and ground is the summation of two components, namely the elastic repulsive force and the dissipative viscous force. These two phenomena are materialized by the parallel association of a spring and damper elements, as it is represented in the scheme of Fig. 4.



Figures 4: Spring-damper model representing the foot-ground contact in the normal and tangential directions

Thus, diving the normal contact force into the elastic and damping components, as in the case of the Kelvin-Voigt model (Goldsimth 1960), yields

$$\mathbf{f}_n = (K\delta^m + c\dot{\delta})\mathbf{n} \tag{11}$$

in which K and c are the spring stiffness and damping coefficient of the penalty approach, δ denotes the relative penetration, $\dot{\delta}$ is the relative velocity of the contacting elements m a non-linear exponent and $\mathbf{n}$ is the normal unit vector of the contact plane. The amount of viscous damping during the contact is controlled by the coefficient c. The energy dissipated will increase as c becomes larger and the kinetic energy will drive the system to damper faster.

The spring stiffness depends on the geometrical and material properties of the contacting elements. For the case of a contact between a sphere i and a plane j, this parameter is given by (Goldsimth 1960)

$$K = \frac{0.424\sqrt{R_j}}{\frac{1-\upsilon_i^2}{\pi E_i} + \frac{1-\upsilon_j^2}{\pi E_j}} \tag{12}$$

where R_i is the sphere radius, υ and E are the Poisson's ratio and Young's elastic modulus associated with each contacting body. This is a constant parameter. Conversely, the damping coefficient varies with the kinematics of the system, that is, it also depends on the contact velocity. A hysteresis damping function can be incorporated in this force model, which represents the dissipated energy during the impact process, such that proposed by Hunt and Crossley (1975)

$$c = \chi \delta^m \tag{13}$$

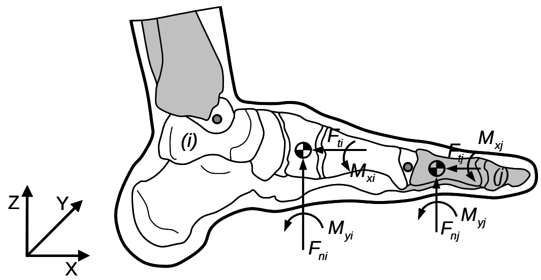
in which χ is denominated the hysteresis damping factor, having the remaining parameters the same meaning as described above.

When the contact between the foot and ground takes place, a friction force must also be incorporated in the contact analysis, in order to account for the shearing force. In the present work, the constitutive law for the tangential force is expressed as

$$\mathbf{f}_t = -(\mu_c f_n + \mu_d v_t)\mathbf{t} \tag{14}$$

which points in the opposite direction of the shearing velocity in the tangential plane of contact. In Eq. (14), μ_c and μ_d represent the dry Coulomb and viscous friction coefficients, f_n is the magnitude of the normal contact force, v_t is the relative tangential velocity and $\mathbf{t}$ is unit vector in the tangential direction.

In short, the normal elastic and damping force components produced in the foot-ground contact is evaluated by using Eqs. (11)-(13). Whilst the tangential force is evaluated by employing Eq. (14), which includes both the dry Coulomb friction and the viscous friction effects. Is should be highlighted that the introduction of the viscous component proved to be quite important, in the measure that it is responsible for preventing the foot from sliding and it is responsible for that dynamic system's response. This aspect plays a crucial role in this type of dynamic problems involving contacts.



Figures 5: Resulting forces and moments that act at the centers of mass of the two foot bodies

It should be noted that during the dynamic simulation of the foot-ground interaction, the number of spheres in contact with the ground surface varies. Thus, the ground vertical and horizontal reaction forces must be evaluated accordingly, that is, as the summation of the contribution of each sphere. Let l represents the number of spheres in contact with ground, then the magnitude of the reaction forces are evaluated

$$F_n = \sum_{k=1}^{l} f_{nk} \tag{15}$$

$$F_t = \sum_{k=1}^{l} f_{tk} \tag{16}$$

where f_{nk} and f_{tk} represent the amount of force developed by each sphere in the normal and tangential direction. The resulting forces are then applied in the center of mass of each body, as it is schematically depicted in Fig. 5. The reaction forces are represented by vectors F_{ni}, F_{ti}, F_{nj} and F_{tj}, for the main foot part and toes, respectively. In addition, the moments that result from the transport of the contact forces are also illustrated in Fig. 5. Finally, the X and Y coordinates of the center of pressure (COP) can be evaluated by using the following expressions

$$X_{COP} = -\frac{M_{yi} + M_{yj}}{F_{ni} + F_{nj}} \quad (17)$$

$$Y_{COP} = -\frac{M_{xi} + M_{xj}}{F_{ni} + F_{nj}} \quad (18)$$

FORMULATION OF THE EQUATIONS OF MOTION FOR MULTIBODY SYSTEMS

This section presents the formulation of the general equations of motion of the spatial dynamic analysis of multibody systems. The present work closely follows the approach developed by Silva (2003). Due to its simplicity and computational easiness, natural coordinates and Newton-Euler's method are used to formulate the equations of motion of the spatial multibody systems (Nikravesh 1988). The methodology presented here can be implemented in any general purpose multibody code, being tested in particular in the computer program APOLLO (Silva 2003), which has been developed for the spatial dynamic analysis of general biomechanical systems. For a constrained multibody system, the kinematical joints are described by a set of algebraic constraint equations denoted as

$$\mathbf{\Phi}(\mathbf{q}, t) = \mathbf{0} \quad (19)$$

Using the Lagrange multipliers technique the constraints are added to the equations of motion. These are written together with the second time derivative of the constraint equations. Thus, the set of equations that describe the motion of the multibody system is

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_\mathbf{q}^T \\ \mathbf{\Phi}_\mathbf{q} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix} \quad (20)$$

where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers and $\boldsymbol{\gamma}$ is the vector that groups all the terms of the acceleration constraint equations that depend on the velocities only,

$$\boldsymbol{\gamma} = -(\mathbf{\Phi}_\mathbf{q}\dot{\mathbf{q}})_\mathbf{q}\dot{\mathbf{q}} - \mathbf{\Phi}_{tt} - 2\mathbf{\Phi}_{\mathbf{q}t}\dot{\mathbf{q}} \quad (21)$$

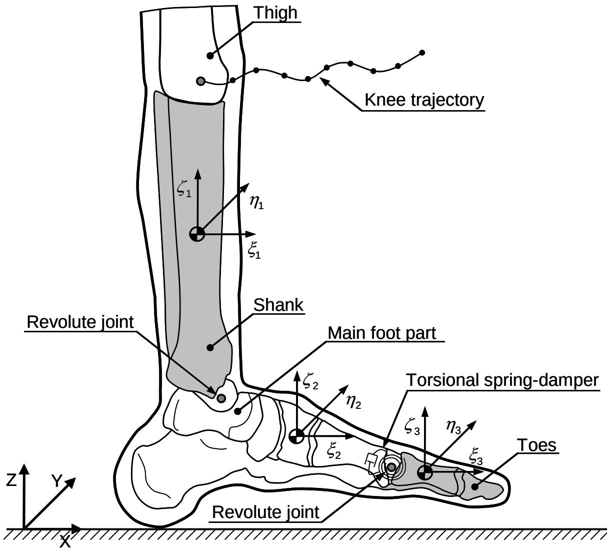
The Lagrange multipliers, associated with the kinematic constraints, are physically related to the reaction forces and moments generated between the bodies interconnected by kinematic joints. Equation (20) is a differential algebraic equation that has to be solved, being the resulting accelerations integrated in time. However, in order to avoid constraints violation during the numerical simulation, the Baumgarte technique is used, being Eq. (20) modified to (Baumgarte 1972)

$$\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_\mathbf{q}^T \\ \mathbf{\Phi}_\mathbf{q} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} - 2\alpha\dot{\mathbf{\Phi}} - \beta^2\mathbf{\Phi} \end{Bmatrix} \quad (22)$$

where α and β are positive constants that represent the feedback control parameters for the velocity and position constraint violations.
In a dynamic analysis, a unique solution is obtained when the constraint equations are considered simultaneously with the differential equations of motion, for a proper set of initial conditions. Thus, mathematically the simulation of constrained multibody system requires the solution of a set of n_c differential equations coupled with a set of m algebraic equations. According to the formulation outlined, the dynamic response of multibody systems involves the evaluation of the Jacobian matrix $\mathbf{\Phi}_\mathbf{q}$ and vectors $\mathbf{g}$ and $\boldsymbol{\gamma}$, each time step. The solution of Eq. (22) is obtained for the system accelerations $\ddot{\mathbf{q}}$. These accelerations, together with the velocities, are integrated to obtain the new velocities $\dot{\mathbf{q}}$ and positions $\mathbf{q}$ for the next time step. This process is repeated until the complete description of system motion is obtained for a selected time interval (Nikravesh 1988).

FOOT MODEL APPLICATION TO HUMAN GAIT ANALYSIS

In this section, some numerical results obtained from some computational simulations of the developed foot model are presented and discussed in order to understand the dynamic behavior of the foot-ground interaction during the human gait. Figure 6 shows a schematic representation of the biomechanical multibody model considered here. In order to keep the analysis simple, this model only includes 4 basic rigid bodies representing the shank, main foot part, toes and ground. Each body has a local coordinate frame located at the center of mass. Two revolute joints are used to connect the main foot part to shank and the main foot part to toes. The main foot part and toes are also constrained by a linear torsional spring-damper element. The torsional spring stiffness is equal to 12 N/m, while the damping coefficient is equal to 0.5 Ns/m. Furthermore, the present study only accounts for the skeletal structure, being the effect of muscles, tendons and ligaments neglected.



Figures 6: Representation of the global biomechanical multibody model

A number of parameters is required by the APOLLO program such as the number of feet involved, the local coordinates of the normal vector to the floor and the number of the spheres used to represent the feet surface. In the present work, for each foot, the maximum number of spheres allowed is equal to 25. The sphere-plan contact geometrical and material properties should be specified by the user, namely local coordinates of the spheres, the sphere radius, coulomb and viscous coefficients, relative stiffness, non-linear degree of the contact law, normal damping coefficient. The proposed foot model allows the evaluation of the ground reaction forces produced during the contact as well as the calculation of the center of pressure position under the plantar surface of the foot. The biomechanical model is based on the data published by Winter (1990), being their anthropometric dimensions those that correspond to a male of 1.70 m and 70 kg. The biomechanical system encompasses 6 functional degrees of freedom: 3 for the knee/shank translational trajectories, 1 for the knee flexion, 1 for the ankle joint rotation and 1 for the metatarsal-phalangeal joint rotation. The mass and moment of inertia properties of each body are listed in Table 1.

In the particular cases studied in this work, the foot and ground interaction is provided by the introduction of a 9 spheres under plantar surface of the foot, 6 located under the main foot part and 3 situated under the toes. The spheres are number from 1 to 9 starting from the most left sphere. The local coordinates of the center of each sphere and their radii are presented in Table 2. As far as the evaluation of the contact forces is concerned, all the spheres have the same properties relative to the contacting stiffness, damping coefficient, degree of nonlinearity and viscous friction coefficient used in the constitutive contact force laws, which values are, respectively, 40 kN/m, 300 Ns/m, 1.5 and 3.0 Ns/m. The friction coefficient is a variable parameter that can vary from 0.5 to 1.0. These values are used to produce the reference data presented in this work.

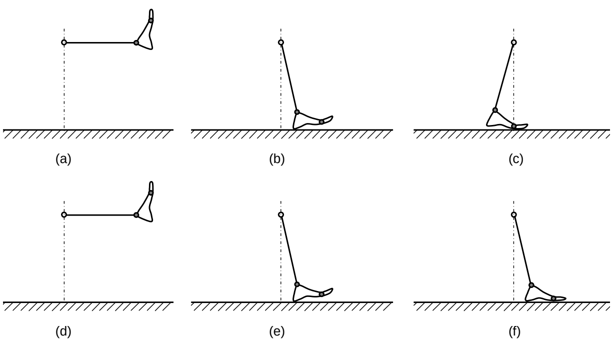
Table 1: Inertia properties of the biomechanical model

Segment	Description	Mass [kg]	Moment of inertia [kgm^2]		
			$I_{\xi\xi}$	$I_{\eta\eta}$	$I_{\zeta\zeta}$
1	Shank	4.76	8.230	8.230	8.230
2	Main foot part	1.33	2.250	2.250	2.250
3	Toes	0.35	0.471	0.471	0.471

Table 2: Geometric properties of the contact spheres

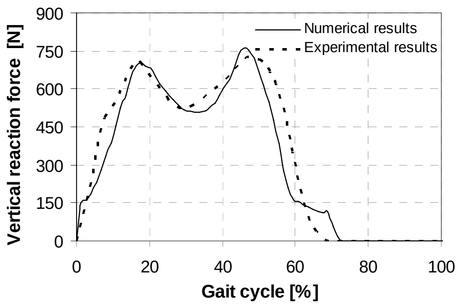
Sphere number	ξ [mm]	η [mm]	ζ [mm]	Radius [mm]
1	-94.70	0.00	-15.34	30.81
2	-94.70	0.00	-15.89	39.53
3	-75.20	0.00	-22.89	31.10
4	0.00	0.00	-25.89	15.10
5	26.97	0.00	-32.36	12.94
6	56.09	0.00	-34.52	15.10
7	-22.70	0.00	-15.10	12.94
8	0.00	0.00	-17.26	10.78
9	21.60	0.00	-12.94	10.79

Prior to analyze the dynamic response of the foot model associated with human gait, it is important to demonstrate the crucial role of the Coulomb and viscous friction effects. For this purpose, two different simulations were performed. In the first case, the viscous effect is neglected, and the static Coulomb friction coefficient is equal to 1.0. In the second case, the biomechanical model is simulated with both Coulomb friction and viscous friction actions. Initially, the model is in a horizontal position as Fig. 7a shows. The system is then released from its initial configuration under gravity action only, which is taken as acting in the negative *Z* direction. When contact between the foot and ground is detected, the contact forces generated are evaluated according to the continuous force model described in the previous section. Figures 7a-c represent the animation sequence corresponding to the first case, that is, when only the Coulomb friction is incorporated in the contact interaction between foot and ground. In turn, Figs. 7d-f show the animation sequence when both Coulomb and viscous friction effects are considered. Based on these illustrations, it can be observed that in the first scenario, the foot describes a rotational motion with higher amplitude, visible when the foot goes beyond the vertical line, as Fig. 7c demonstrates. In sharp contrast are the representations of the second scenario, in which the foot stops before the vertical line. This behavior can be explained due to the incorporation of the viscous friction effect. Finally, it can be observed the key role played by the viscous friction, which is in fact much more realistic.



Figures 7: (a)-(c) Animation sequence with only Coulomb friction; (d)-(f) Animation sequence with both Coulomb and viscous friction effects

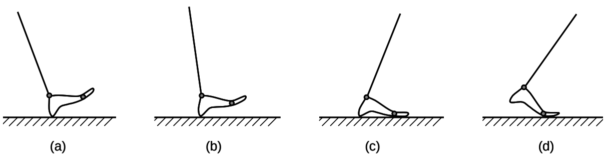
In what follows, some obtained results from a computational simulation corresponding to a forward dynamic analysis of the biomechanical model described above are presented and discussed. The global motion is relative to a complete gait cycle. The analysis is performed considering the full prescribed kinematic data, that is, all the functional degrees of freedom are guided through the entire gait cycle. The global dynamic response of the system is quantified by plotting the vertical ground reaction force. The computational results for the vertical reaction force are compared to the data obtained experimentally, as it can be observed in the diagrams of Fig. 8. In general, there is a good agreement between computational and experimental approaches, being also supported by some the best published literature in this particular filed of investigation (Silva 2003), namely in what concerns with the vertical reaction force. From the plot of the center of pressure it is possible to quantify the foot length and also the evolution of the foot path during the gait cycle.



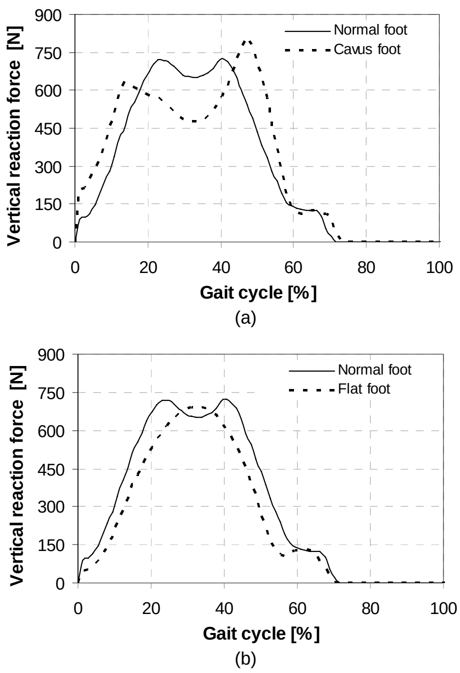
Figures 8: Vertical reaction force

The global motion produced by the biomechanical multibody model is illustrated in the animation sequence of Fig. 9, where the main phases of the human gait are presented, namely, the swing and the stance phases. In the case of the stance phase, three sub-phases can be identified, that is, the heel contact (initial contact), the plantar support (mid stance) and toe off (terminal stance). Figure 9a shows the swing phase (terminal swing), which starts when the shank is vertical and ends when the foot touches the ground. In the phase there is no contact between the foot and the ground, hence there is no reaction forces. The three sub-phases of the stance period can be describes as follows. The stance phase itself starts with the initial contact between the foot and ground, as it is depicted in Fig. 9b. In general, the heel is the first part to touch the ground (heel contact). In the sub-phase the spheres 1 to 3 contact with ground, being this situation corresponding to 10% of stance phase. Figure 9c shows the mid stance sub-phase, in which the body weight is transferred to the leg. This sub-phase is of utmost important for shock absorption, weight-bearing and forward progression. In the mid stance sub-phase, spheres 1 to 8 contact the ground, which corresponds to 49% of stance phase. Finally, the end of stance phase is visible in the representation of Fig. 9d. This last sub-phase begins when the toes are the only part of the foot to contact the ground, which is usually denominated as the toe off. In this sub-phase, the spheres 7 to 9 contact the ground that represents 68% of stance phase. To sum up, it can be said that the global behavior of the contact foot model is realistic and corroborates the data available (Debrunner and Hepp 1999, Gefen 2003).

Finally, in order to demonstrate the application of the proposed methodology in the study of the influence of the shape of the plantar surface on the response of the system, two different plantar surface pathologies are considered, namely one that representing a cavus foot and one corresponding to a flat foot pathologies. These two pathologies are associated with anomalies on topography of plantar surface of the foot. The cavus foot pathology is characterized by arches with much higher amplitude than usual and is caused by weakness in the muscles, especially muscles in the leg and feet, can cause this pathology, or due to nerve or muscle disease. These unbalanced muscles, especially in the foot and ankle, work unevenly causing the high arch. This pathology, eventually, causes pain in the patient and they have high probability to develop thick calluses under the metatarsal-phalangeal joints. Flat foot or pronated foot is a common foot shape. This pathology is characterized by the absence of the arch of the foot. When a patient suffering from this pathology stands up, the arch in the middle of the foot disappears and the foot seems to lie flat on the ground.



Figures 9: Animation sequence of the human gait cycle of the biomechanical model



Figures 10: Vertical reaction force: (a) Cavus foot; (b) Flat foot

The influence of the different foot pathologies on the reaction force is illustrated in the diagrams of Fig. 10 where their plots are compared with the data corresponding to a foot without pathology. In the case of the cavus foot, the vertical ground reaction force produced is higher during the heel contact, at the end of mid stance and toe off phases, and presents lower amplitude during the mid stance phase. Indeed, for this foot pathology, the contact is more intensive in the heel phase because the body weight is more concentrated in the heel and toes, and is less intensive in the central zone in which the cavus pathology is located. On the other hand, in the case of flat foot pathology is can be drawn that the contact force evolution tends to have a similar shape to the classic Hertz contact.

CONCLUSIONS

A comprehensive three-dimensional model of the interaction developed between the foot and ground during human gait has been presented and discussed in this work. The proposed model is developed under the framework of multibody systems methodologies. In a simple way, the foot and ground are modeled as contacting elements, which characteristics are function of the geometric and material properties of the contacting surfaces. The interaction between the proposed foot model and the ground is modeled using sphere-plan contact pairs that can be adjusted in number, size and position in order to ensure a better replication of the foot plantar surface.

For each contact pair a force is calculated based in the kinematic characteristics of the contact, namely relative pseudo-penetration and on the relative material characteristics of the surfaces in contact. More precisely, a continuous viscoelastic contact force model used is the one the Kelvin-Voigt model (for the normal-to-the-ground component of the contact force) and on a dry-friction model (for the tangential-to-the-ground components of the contact force). With respect to the Kelvin-Voigt model, the force produced by the spring component is calculated according to the Hertzian contact theory while the force produced in the viscous element follows Hunt and Crossley hysteretic approach. The friction force model presents one of the most important and novel features of the proposed foot model as it includes two distinct components: a standard Coulomb friction component and a viscous friction component that is proportional to the tangential-to-the ground velocity of foot. Furthermore the proposed methodology was implemented in a three-dimensional multibody code, named APOLLO that is able to automatically generate the equations of motion of any biomechanical system and to solve them for forward and inverse dynamics.

Some numerical results obtained from different computational simulations are presented in order to discuss the methodologies and premises adopted through this work. In short, the three-dimensional computational foot model here proposed is a reliable reproduction of contact between foot and floor. It was demonstrated that, when contact occurs, the Coulomb friction force alone was not able to stop the foot from an excessive sliding even when the Coulomb friction coefficient was set to one. This result is of crucial importance and may justify the reason why several reviewed models fail to duplicate the proper kinematics and, consequently, the dynamics of the foot-ground interaction during the simulation of a normal gait cycle. With this foot contact model, the use of a force platform to evaluate the ground reaction forces could be dispensable in human gait analysis tasks. The numerical results obtained from several computational simulations were compared to those published in the literature.

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