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OPTIMIZATION METHODS FOR TWO-DIMENSIONAL CUTTING STOCK AND BIN PACKING PROBLEMS

Elsa Silva[ac]
Filipe Alvelos[abc]
José Manuel Valério de Carvalho[abc]

Algoritmi Research Center[a]
Systems and Production Department[b]
University of Minho[c]
{elsa, falvelos, vc}@dps.uminho.pt

We propose new solution approaches for three problems: the cutting stock problem, the integrated cutting stock and lot sizing problem, and the bin packing problem. In all cases, we consider two dimensional problems.

Cutting stock

In a two-dimensional cutting stock (*2DCS*) problem, it is intended to cut a set of rectangular items from a set of rectangular plates in such a way that the number of used plates is minimized. The plates are available in (a virtually) infinite number and all have the same dimensions, i.e., the same width and height.

A set of items to be cut, grouped by types according to their dimensions (width and height), is given. Each item type is defined by a width, a height and a demand (corresponding to the number of items of the type to be cut). This problem is equivalent to the two-dimensional bin packing problem, in which it is intended to place the set of items in the plate (bin) with the same goal.

The 2DCS problem and its variants arise in different industrial processes, such as the textile, the leather, the paper, the furniture and the sheet metal. This problem has been extensively studied in the last fifty years, due to its wide industrial applicability, however there are not general and efficient methods to solve it due to its computational complexity (NP-hard).

The motivation behind this work lies in the woodcut industry where plates of wood must be cut in pieces (items) to satisfy customer orders. In this industry, due to technological constraints, usually only cuts parallel to the sides of the plate (orthogonal cuts) and from one border to the opposite one (guillotine cuts) are allowed. Furthermore, the number of stages (set of cuts with the same orientation horizontal or vertical) in which a plate can be cut is also frequently limited to two or three.

An approach to obtain optimal solutions to different variants of the 2DCS problems, such as the existence of constraints on the type and number of cuts (two-stage and three-stage exact and non-exact) which is not based on column generation, is proposed.

The model is an extension of the one-cut model for the one-dimensional cutting stock problem proposed by Dyckhoff (1981). The approach is based on an integer programming model that can be solved by a general integer programming solver, taking advantage of the efficiency and robustness that (mixed-) integer programming solvers have achieved in recent years. Although pseudo-polynomial in size, the model optimized by a general-purpose integer programming solver (Cplex 11.0) solved a large number of “real-world” instances in a very reasonable amount of time.

Integrated cutting stock and lot sizing

The *2DCS* consists in the minimization of the number of plates used to cut a set of items. In industry, typically, an instance of this problem is considered at the beginning of each planning period, what may result in solutions of poor quality, i.e, excessive waste, when a set of planning periods is considered. In order to deal with this issue, we propose the integrated problem, in which the 2DCSP is extended from the solution in only a single production planning period to a solution in a set of production planning periods.

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The main difference of the approach in this work and the ones in the literature is to allow sufficiently large residual plates (leftovers) to be stored and cut in a subsequent period of the planning horizon, which may further help in the minimization of the waste.
We propose two integrated integer programming models to optimize the combined two dimensional cutting stock and lot-sizing problem, minimizing the waste, the storage costs and the number of plates used.

Two new exact models were presented, based on different decision variables, that allow to store items and leftovers (and also complete plates), and so leftovers can be used in a subsequent period in a two-stage non-exact cutting pattern.

The two integer programming models proposed are quite similar regarding the computational results obtained. The real instances were solved optimally in acceptable computational time and comparing these solutions with the heuristic ones, it is observed that when the number of periods increases the gains obtained by the optimal solutions also increase.

The proposed models are developed for a variant of the 2CS-LSP in which both the anticipation of the production of the items and the storage of leftovers are allowed, with the objective of minimizing production and storage costs. In the literature this issue has never been addressed, but there are some similar problems. Besides presenting an approach that may provide better solutions in terms of the use of the raw material, it is important to notice that we were able to obtain exact solutions for our model, while in the previous works only heuristic results were obtained for instances of similar size.

Bin packing

The two dimensional bin packing problem (*2BP*) is quite similar with the *2DCS* problem, the main difference is the assortment of the items, which is strongly heterogeneous, meaning that all items are different and the demand is equal to one. A general framework amenable for decomposition approaches "SearchCol", which combines column generation with metaheuristic search, is used to solve the *2BP*.

SearchCol is a general approach to obtain approximate solutions for combinatorial optimization problems. In this framework a solution of a problem is a combination of solutions of smaller (sub)problems SPs which are generated by column generation (*CG*). The search is conducted in restricted search spaces, defined by the solutions of the SPs generated by *CG*.

Each iteration of SearchCol has three phases: firstly the CG, secondly the metaheuristic search and thirdly the perturbation. A decomposition model and new CG algorithm were proposed for the application of SearchCol in the (*2BP*) problem. Interesting results were obtained with SearchCol, in particular when MIP is used in the search phase. The computational tests were performed in a set of 500 instances from the literature.

AUTHOR BIOGRAPHIES

Elsa Silva has graduated in Applied Mathematics from University of Minho in 2007. Currently, she is working in her Ph.D. thesis, her main research interests are exact methods and heuristics for two dimensional cutting stock and bin packing problems. Her e-mail address is: elsa@dps.uminho.pt

Filipe Alvelos is Associate Professor at the Department of Production and Systems Engineering, of University of Minho, Portugal. He is also a researcher of the Systems Engineering, Optimization and Operations Research Group of the Algoritmi Research Center. His current research interests are exact and heuristic methods for integer and combinatorial optimization with applications to network design and routing, production planning and scheduling, and cutting and packing problems.

J. M. VALÉRIO DE CARVALHO is Professor of Operations Research in the University of Minho, and researcher of the Systems Enginee-ring, Optimization and Operations Research (SEOOR) Group of the Algoritmi Research Center. His main research interests include large scale integer programming, and applications in cutting and packing problems, scheduling and network design. His e-mail address is: vc@dps.uminho.pt and his Web-page can be found at http://pessoais.dps.uminho.pt/vc/.