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Resource Complementarity in Activity Networks

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ABSTRACT

In this paper we address the issue of optimal resource allocation, more specifically, the resource complementarity analysis in a project activity networks. We developed a conceptual system capable of determining the ideal mixture of resources allocated to the activities of a project, such that the project is completed on time with minimal cost.

The results of computational implementation will be exposed in detail in the full version of this paper.

INTRODUCTION

This paper is concerned with the optimal resource allocation in activity networks under conditions of resource complementarity. Aspects related to performance improvement, short duration, quality improvement have been presented by (Silva et al. 2010) as well as the effect of the “supportive” resource for the project cost. Some preliminary results have been already presented (Elmaghraby and Girish 2010; Silva et al. 2010; Silva et al. 2010a).

PROBLEM DESCRIPTION

Consider a project network in the activity-on-arc (AoA) representation: $G = (N, A)$ with the set of nodes $|N| = n$ (representing the “events”) and the set of arcs $|A| = m$ (representing the “activities”). In general each activity requires the simultaneous use of several resources (Tereso et al. 2008; Tereso et al. 2009a; Tereso et al. 2009b) .

There is a set of “primary” resources, denoted by P, with $|P| = \rho$. Additionally, there is a pool of “support” resources, denoted by S, with $|S| = \sigma$ (such as less-skilled labor, or computers and electronic devices; etc.) that may be utilized in conjunction with the primary resources to enhance their performance.

The number of support resources varies with the activity and the primary resources required for its execution. The applicability and impact of support resources is the same as presented by (Silva et al. 2010).

If only one unit of S-resource is used, the performance of the allocation of P-resource r_p to activity a, which is denoted by $x_a(r_p)$, is augmented to,

$$x_a(r_p, s_p) = x_a(r_p) + v(r_p, s_p) \tag{1}$$

Once that the impact of the S-resources is additive: if a subset $\{s_q\}_{q=1}^{\sigma}$ of the S-resources is used in support of P-resource r_p in activity a, and only one unit of each S-resource is used, then the performance of the former is enhanced to,

$$x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right) = x_a(r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q) \tag{2}$$

The primary resource $r_p \in P$ would accomplish activity a in time $y_a(r_p)$. If it is enhanced by the addition of one S-resource s_q then its processing time decreases to $y_a(r_p, s_q)$, with $y_a(r_p, s_q) < y_a(r_p)$.

The duration of activity a when using resource r_p is given by (Tereso et al 2004),

$$y_a(r_p) = \frac{w_a(r_p)}{x_a(r_p)} \tag{3}$$

If a support resource s_q is added to the primary resource r_p then the duration becomes,

$$y_a(r_p, s_q) = \frac{w_a(r_p)}{x_a(r_p, s_q)} \tag{4}$$

MATHEMATICAL MODEL

We assume that all costs are linear or piece-wise linear in their argument.

Let:

C^k*:* the *k*th uniformly directed cutset *(udc)* of the project network $k = 1, \dots, K$.

$x_a(r_p)$*:* level of allocation of (primary) resource r_p to activity a.

$x_a^{r_p}(s_q)$: level of allocation of secondary resource s_q to primary resource r_p in activity a.

$x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right)$: total allocation of resource r_p (including complementary resources) to activity a.

$v(r_p, s_q)$: degree of enhancement of P-resource r_p by S-resource s_q.

$w_a(r_p)$: work content of activity a for P-resource r_p.

$y_a(r_p, \{s_q\}_{q=1}^{\sigma})$: duration of activity a imposed by primary resource r_p and resources s_q.
$y(a)$: duration of activity a (considering all resources).
ρ : number of primary resources, $\rho = |P|$.
σ : number of secondary resources, $\sigma = |S|$.
$Q_P(p)(Q_S(q))$: capacity of P-resource r_p (S-resource s_p) available.
γ_p : marginal cost of P-resource r_p.
γ_q : marginal cost of S-resource s_q.
γ_E : marginal gain from early completion of the project.
γ_L : marginal loss (penalty) from late completion of the project.
t_i : time of realization of node i (AoA representation), where node 1 is the "start node" of the project and node n its "end node".
T_s : target completion time of the project (due date).
$c_R(a, r_p)$: cost of resources for activity a resource r_P (including complementary resources).
$c_R(a)$: cost of resources for activity a (includes all resources).
e : earliness.
d : tardiness (delay).
c_E : cost of earliness.
c_T : cost of tardiness.
c_{ET}: cost of earliness and tardiness.
TC: total cost.

The notation $a \equiv (i, j)$ means that activity a is represented by arc (i, j).

Respect precedence among the activities:

$$t_j \geq t_i + y(a), \quad \forall a \equiv (i, j) \in A \qquad (5)$$

The total allocation of resource r_p in activity a is,

$$x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right) = x_a(r_p) + \sum_{q=1}^{\sigma} v(r_p, s_q) * x_a^{r_p}(s_q) \qquad (6)$$

The duration is defined by:

$$y_a(r_p, \{s_q\}_{q=1}^{\sigma}) = \frac{w_a(r_p)}{x_a\left(r_p, \{s_q\}_{q=1}^{\sigma}\right)} \qquad (7)$$

$$y(a) = \underbrace{max}_{all\ r_p}\left\{y_a(r_p, \{s_q\}_{q=1}^{\sigma})\right\} \qquad (8)$$

Respect the P-resource availability at each *udc*[1],

$$\sum_{a \in C^k} x_a(r_p) \leq \mathrm{Q_P(p)}, \qquad \forall p \in P \qquad (9)$$

in which $Q(p)$ is the capacity (i.e., availability) of P-resource r_p and respecting also the S-resources availability, we have

$$\sum_{a \in C^k} x_a^{r_p}(s_q) \leq \mathrm{Q_S(q)} \quad \forall q \in S \qquad (10)$$

in which $Q_S(q)$ is the capacity of S-resource s_q (in the three-activities.

Define earliness and tardiness by,

$$e \geq \mathrm{T}_s - t_n \qquad (11)$$

$$d \geq t_n - \mathrm{T}_s \qquad (12)$$

$$e, d \geq 0 \qquad (13)$$

The criterion function is composed of two parts: the cost of use of the P- and S-resources, and the gain or loss due to earliness or tardiness, respectively, of the project completion time t_n relative to its due date.

The cost of resource utilization is quadratic in the resource allocation for the duration of the activity (Tereso et al 2004),

$$c_R(a, r_p) = \left(\gamma_p * x_a(r_p) + \gamma_q * \sum_{q=1}^{\sigma} x_a^{r_p}(s_q)\right) * w(a, r_p) \qquad (14)$$

$$c_R(a) = \sum_{\text{all } \mathrm{r_p}} c_R(a, r_p) \qquad (15)$$

(ii) The earliness-tardiness costs are linear in their respective marginal values, as shown in eq. (16)

$$c_{ET} = c_E + c_T = \gamma_E \cdot e + \gamma_L \cdot d \qquad (16)$$

$$\min TC = \sum_{a \in A} c_R(a) + c_{ET} \qquad (17)$$

CONCLUSION

The relevance of the problem is the opportunity to shape a system that allows not only that we improve the allocation of often scarce resource(s), but also result in reduced uncertainties within the projects, combined with increased performance and lower project costs. The model was first presented in (Silva et al. 2010) and some preliminary results were presented in (Silva et al. 2010a) and (Silva et al. 2010b) but there remained its implementation and application to a larger set of project networks, to demonstrate its validity. In this paper we only presented the mathematical model, but a procedure was already developed to solve the mathematical model and it was applied to a test set of activities networks. A general computer code was also developed and is now capable to solve any kind of activity network problem.

We believe it can provide to user a new option to plan and to determine the best combination of resources and the lowest project cost, pushing the planning phase and increase the estimation ability of the companies.

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[1] The acronym *udc* stands for 'uniformly directed cutset', which is a cutset of the graph in which all arrows are directed from the subset of nodes H which contains the origin node, to the complementary subset $\overline{H} = N - H$ which contains the terminal node.

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